qpkit — Sample Paper

CBSE Class X Board Pattern

Class: X | **Subject: Mathematics (Standard)** | **Time: 3 hours** | **Max. Marks: 80**

SECTION A — MULTIPLE CHOICE QUESTIONS

1. Find the coordinates of the midpoint of the line segment joining $(2,\ -2)$ and $(-8,\ -4)$. **1**

Ans: $(-3,\ -3)$

Working: $\left(\frac{2+-8}{2},\ \frac{-2+-4}{2}\right) = (-3,\ -3)$.

2. The quadratic equation $4x^2 - 7x + 10 = 0$ has **1**

(A) two distinct real roots (B) two equal real roots

(C) no real roots (D) more than two real roots

Ans: (C)

Working: $b^2 - 4ac = -111$.

3. Find the nth term of the AP $0,\ 7,\ 14,\ 21,\ \ldots$ for $n = 26$. **1**

Ans: 175

Working: $a_n = a + (n-1)d = 0 + 25 \times (7) = 175$.

4. Find the product of the zeroes of the polynomial $x^2 - 1$. **1**

Ans: -1

Working: Product of zeroes $= \dfrac{c}{a} = -1$.

5. Tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 66°. Find $\angle$ POA. **1**

Ans: 57°

Working: $\angle$ AOB $= 180^\circ - 66^\circ$, and OP bisects it, so $\angle$ POA $= 57^\circ$.

6. The angle of elevation of the top of a tower from a point on the ground, 45 m away from its foot, is 30°. Find the height of the tower. **1**

Ans: $15\sqrt{3}$ m

Working: $h = 45 \tan 30^\circ = 15\sqrt{3}$ m.

7. If $\tan A = \frac{12}{5}$ for an acute angle A, find $\sin A$. **1**

Ans: $\frac{12}{13}$

Working: The sides are $12, 5, 13$, so $\sin A = \frac{12}{13}$.

8. Which of the following equations has 4 as a root? **1**

(A) $x^2 - 36 = 0$ (B) $3x^2 - 48 = 0$

(C) $2x^2 + 6x = 0$ (D) $x^2 - 10x + 25 = 0$

Ans: (B)

Working: Substituting $x = 4$ makes only option (B) vanish.

9. The zeroes of the polynomial $x^2 + 8x$ are **1**

(A) $-7,\ -1$ (B) $-8,\ 0$

(C) $0,\ -7$ (D) $8,\ 0$

Ans: (B)

Working: Factorising gives zeroes -8 and 0.

10. The pair of linear equations $3x + 6y = 14$ and $6x + 12y = 28$ has **1**

(A) exactly one solution (B) no solution

(C) infinitely many solutions (D) exactly two solutions

Ans: (C)

Working: The coefficient ratios give the coincident case.

11. A bag contains 11 green balls and 3 blue balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is green? **1**

Ans: $\frac{11}{14}$

Working: $P = \dfrac{11}{14} = \frac{11}{14}$.

12. The distance between the points $(5,\ -7)$ and $(13,\ 8)$ is **1**

(A) 23 units (B) 289 units

(C) 7 units (D) 17 units

Ans: (D)

Working: $\sqrt{8^2 + 15^2} = 17$.

13. Find the discriminant of the quadratic equation $x^2 - 13x + 1 = 0$. **1**

Ans: 165

Working: $b^2 - 4ac = (-13)^2 - 4(1)(1) = 165$.

14. Find the distance between the points $(6,\ -7)$ and $(1,\ 5)$. **1**

Ans: 13 units

Working: $\sqrt{(5)^2 + (12)^2} = 13$.

15. The discriminant of the quadratic equation $4x^2 + 2x - 1 = 0$ is **1**

(A) 8 (B) -12

(C) -20 (D) 20

Ans: (D)

Working: $b^2 - 4ac = (2)^2 - 4(4)(-1) = 20$.

16. State whether the pair of linear equations $8x + 4y = 15$ and $16x + 8y = 34$ is consistent or inconsistent. **1**

Ans: Inconsistent (lines are parallel)

Working: Comparing $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ gives the parallel case.

17. Find the sum of the zeroes of the polynomial $2x^2 - 5x - 7$. **1**

Ans: $\frac{5}{2}$

Working: Sum of zeroes $= -\dfrac{b}{a} = \frac{5}{2}$.

18. The distance between the points $(-5,\ 7)$ and $(2,\ 31)$ is **1**

(A) 17 units (B) 625 units

(C) 31 units (D) 25 units

Ans: (D)

Working: $\sqrt{7^2 + 24^2} = 25$.

19. **Assertion (A):** The distance between $(-4,\ -4)$ and $(3,\ 20)$ is 25 units. **1**
Reason (R): The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Ans: (A)

Working: $\sqrt{7^2 + 24^2} = 25$, straight from (R).

20. **Assertion (A):** The distance between $(6,\ 2)$ and $(13,\ 26)$ is 25 units. **1**
Reason (R): The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Ans: (A)

Working: $\sqrt{7^2 + 24^2} = 25$, straight from (R).

SECTION B — VERY SHORT ANSWER QUESTIONS

21. E and F are points on the sides PQ and PR respectively of $\triangle$ PQR. Given PE $= 16$ cm, EQ $= 7.5$ cm, PF $= 5.5$ cm and FR $= 9$ cm, state whether EF $\|$ QR. Justify your answer. **2**

Ans: No, because $\dfrac{\text{PE}}{\text{EQ}} \neq \dfrac{\text{PF}}{\text{FR}}$

Working: $\dfrac{\text{PE}}{\text{EQ}} = \frac{32}{15}$ and $\dfrac{\text{PF}}{\text{FR}} = \frac{11}{18}$.

22. A rope 30 m long is tightly stretched from the top of a vertical pole to a peg on the ground, making an angle of 30° with the ground. Find the height of the pole. **2**

Ans: 15 m

Working: $h = 30 \sin 30^\circ = 15$ m.

23. In $\triangle$ ABC, right-angled at B, AB $= 112$ cm and BC $= 66$ cm. Determine $\sin$ A, $\cos$ A and $\tan$ A. **2**

Ans: $\sin A = \frac{33}{65}, \cos A = \frac{56}{65}, \tan A = \frac{33}{56}$

Working: AC $= \sqrt{112^2 + 66^2} = 130$ cm, and the ratios follow.

24. Find the LCM and HCF of 363 and 663, and verify that LCM $\times$ HCF $=$ product of the two numbers. **2**

Ans: LCM $= 80223$, HCF $= 3$

Working: $363 = 3 \times 11^2, 663 = 3 \times 13 \times 17; 80223 \times 3 = 363 \times 663 = 240669$.

25. Find the area of a quadrant of a circle whose circumference is 264 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **2**

Ans: $1386\ \text{cm}^2$

Working: $2\pi r = 264$ gives $r = 42$ cm, so the quadrant is $1386\ \text{cm}^2$.

SECTION C — SHORT ANSWER QUESTIONS

26. In $\triangle$ ABC, D and E are points on the sides AB and AC respectively such that DE $\parallel$ BC. Prove that $\text{AD} \times \text{EC} = \text{AE} \times \text{DB}$. Hence, if AD $= 16$ cm, DB $= 4$ cm and AE $= 9$ cm, find EC. **3**

Ans: $\text{EC} = 2.25$ cm

Working: Since DE $\parallel$ BC, $\dfrac{\text{AD}}{\text{DB}} = \dfrac{\text{AE}}{\text{EC}}$ by the Basic Proportionality Theorem. Cross-multiplying gives $\text{AD} \times \text{EC} = \text{AE} \times \text{DB}$. Substituting, $\text{EC} = \dfrac{9 \times 4}{16} = 2.25$ cm.

27. From an external point A, two tangents AB and AC are drawn to a circle with centre O, touching it at B and C. Prove that $\angle$ BAC $+ \angle$ BOC $= 180^\circ$. Hence, if $\angle$ BAC $= 110^\circ$, find $\angle$ BOC. **3**

Ans: $\angle$ BOC $= 70^\circ$

Working: OB and OC are perpendicular to the tangents at B and C, so in the quadrilateral OBAC two angles are 90° each. The angles of a quadrilateral sum to 360°, so $\angle$ BAC $+ \angle$ BOC $= 180^\circ$, giving $\angle$ BOC $= 70^\circ$.

28. Given $\cot\theta = \frac{56}{33}$, calculate all the other trigonometric ratios of θ. **3**

Ans: $\sin\theta = \frac{33}{65}, \cos\theta = \frac{56}{65}, \tan\theta = \frac{33}{56}, \cot\theta = \frac{56}{33}, \sec\theta = \frac{65}{56}, \text{cosec}\,\theta = \frac{65}{33}$

Working: The right triangle has sides 33, 56 and hypotenuse 65.

29. Two concentric circles are of radii 75 cm and 60 cm. Find the length of the chord of the larger circle which touches the smaller circle. **3**

Ans: 90 cm

Working: The perpendicular from the centre bisects the chord, so half of it is $\sqrt{75^2 - 60^2} = 45$ cm and the chord is 90 cm.

30. From the top of a building 45 m high, the angle of depression of a car is 30°. Find the horizontal distance of the car from the foot of a building. **3**

Ans: $45\sqrt{3}$ m

Working: The angle of depression equals the angle of elevation from the car, so the distance is $\dfrac{45}{\tan 30^\circ} = 45\sqrt{3}$ m.

31. From an external point P, two tangents PA and PB are drawn to a circle of radius 72 cm with centre O. If OP $= 90$ cm, find the length of each tangent and the perimeter of the quadrilateral OAPB. **3**

Ans: Each tangent 54 cm; perimeter 252 cm

Working: PA $=$ PB $= \sqrt{90^2 - 72^2} = 54$ cm, so the perimeter is $2(72 + 54) = 252$ cm.

SECTION D — LONG ANSWER QUESTIONS

32. A lot of 20 pens contains 6 defective ones. (i) One pen is drawn at random from the lot. What is the probability that this pen is defective? (ii) Suppose the pen drawn in (i) is not defective and is not replaced. Now one pen is drawn at random from the rest. What is the probability that this pen is not defective? **5**

Ans: (i) $\frac{3}{10}$ (ii) $\frac{13}{19}$

Working: (i) $\dfrac{6}{20} = \frac{3}{10}$. (ii) After removing one good pen, $\dfrac{13}{19} = \frac{13}{19}$.

33. A contract on a construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹250 for the first day, ₹350 for the second day, ₹450 for the third day, and so on, the penalty for each succeeding day being ₹100 more than for the preceding day. How much money must the contractor pay as penalty if the work is delayed by 25 days? **5**

Ans: ₹36250

Working: $a = 250$, $d = 100$, $n = 25$, so $S_{25} = 36250$.

34. Two water taps together can fill a tank in $14\frac{2}{5}$ hours. The tap of larger diameter takes 12 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **5**

Ans: Larger tap 24 hours, smaller tap 36 hours

Working: $\dfrac{1}{y} + \dfrac{1}{y+12} = \dfrac{1}{\frac{72}{5}}$ gives $y = 24$.

35. The following frequency distribution gives the weights of students of a class. Find the mean, median and mode of the data and compare them. **5**

Weight (in kg)	Number of students
40-45	23
45-50	24
50-55	7
55-60	12
60-65	4
65-70	16
Total	86

Ans: Mean $= 52.38$, Median $= 49.17$, Mode $= 45.28$

Working: Mean 52.38, median 49.17, mode 45.28; they satisfy Mode ≈ 3 Median $- 2$ Mean approximately.

SECTION E — CASE STUDY BASED QUESTIONS

36. For the temple festival procession, two groups of volunteers have to march in columns. The first group has 420 volunteers and the second has 196. Every column must hold the same number of volunteers, and no group may be split between columns. **4**

(i) Express 420 as a product of its prime factors. 1

Ans: $2^2 \times 3 \times 5 \times 7$

(ii) Find the largest number of volunteers that a column can hold if both groups march in columns of that size. 1

Ans: 28

(iii) The organiser suggests columns of 7 volunteers. Is that possible for both groups? Justify your answer. 2

Ans: Yes, 7 divides the HCF 28

Working: $420 = 28 \times 15$ and $196 = 28 \times 7$, and $\gcd(15, 7) = 1$, so the HCF is 28. A column size works for both only if it divides 28; 7 does.

37. A ground-level water reservoir serving a village is built as a cylinder of radius 10.5 m and height 13 m, with a hemispherical dome of the same radius fixed on top. The panchayat needs to know how much water it holds and how much paint the outside will take. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **4**

(i) Write the formula for the volume of the hemispherical dome. 1

Ans: $\frac{2}{3}\pi r^3$

(ii) Find the volume of the cylindrical part of the tank. 1

Ans: 4504.5 m^3

(iii) Find the total capacity of the tank. 2

Ans: 6930 m^3

Working: $\pi r^2 h + \frac{2}{3}\pi r^3 = 6930 \text{ m}^3$.

38. A craftsperson is making a circular rangoli design of radius 10.5 cm. The circle is divided into 4 equal sectors by straight lines drawn from the centre, and each sector is to be filled with coloured powder of a different colour. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **4**

(i) Find the angle of each sector at the centre. 1

Ans: 90°

(ii) Find the area of one sector. 1

Ans: $86.625\ \text{cm}^2$

(iii) Find the total area of the design and the length of the circular border around it. 2

Ans: $346.5\ \text{cm}^2$ and 66 cm

Working: Each sector has angle 90°, so its area is $86.625\ \text{cm}^2$.

— END OF ANSWER KEY —